VECTOR FIELD RBF INTERPOLATION ON A SPHERE

Michal Smolik, Vaclav Skala
Faculty of Applied Sciences, University of West Bohemia
Univerzitni 8, CZ 30614 Plzen, Czech Republic

ABSTRACT

This paper presents a new approach for Radial Basis Function (RBF) interpolation on a sphere. Standard approaches use the Euclidian metrics for the distance calculation of two points. However, for interpolation on a sphere, more naturally is computation of the distance as the shortest distance over the surface on a sphere, i.e. spherical distance of two points is more natural for interpolation on a sphere. We present the results on synthetic and real wind vector datasets on a globe.

KEYWORDS

Vector field, Radial Basis Functions, interpolation on sphere, visualization, spherical distance.

1 INTRODUCTION

Interpolation is probably the most frequent operation used in computational methods. Several methods have been developed for data interpolation, but they expect some kind of data "ordering". Usually, in technical applications, the scattered data are tessellated using triangulation, but this approach is quite prohibitive for the case of k-dimensional data interpolation because of the computational cost.

Interpolating scattered vector data on a surface becomes frequent in applied problem solutions [Turk, G., O'Brien, J.F., 2002]. When the underlying manifold is a sphere, there are applications to geodesy [Aguilar, F. J., et al, 2005], meteorology [Eldrandaly, K. A., Abu-Zaid, M. S., 2011], astrophysics, geophysics, geosciences [Flyer, N. et al, 2014], and other areas. Radial basis function interpolation on a sphere [Golitschek, M. V., Light, W. A., 2001], [Baxter, B. J., Hubbert, S., 2001] has the advantage of having a continuous interpolant all over the sphere, as there are no borders.

2 RADIAL BASIS FUNCTIONS ON A SPHERE

Radial basis functions (RBF) is a technique for scattered data interpolation [Pan, R. and Skala, V., 2011] and approximation [Fasshauer, G.E., 2007].

Radial basis function interpolation can be computed on a sphere and has some advantages. There are no non-physical boundaries and there are no problems with interpolation on the poles, i.e. the sphere has no boundaries, and the vector field can be interpolated on the whole sphere surface at once. The other advantage is that there are no coordinate singularities and the maximal distance of any two points has an upper limit.

The calculation of the distance r between two points $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ on a sphere can be computed as the Euclidian distance between these two points

$$r = \|\boldsymbol{x}_1 - \boldsymbol{x}_2\| = \sqrt{(\boldsymbol{x}_1 - \boldsymbol{x}_2)^T \cdot (\boldsymbol{x}_1 - \boldsymbol{x}_2)}\,. \tag{1}$$

In cases where both points lie on a unit sphere, then $r \in \langle 0; 2\rangle$.

Another possibility is to compute the distance as the shortest distance between two points $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ on the surface of a sphere, measured along the surface of the sphere. The distance is computed using

$$r = \|\boldsymbol{x}_1 - \boldsymbol{x}_2\|_{spherical} = \cos^{-1}(\boldsymbol{n}_1 \cdot \boldsymbol{n}_2)\,, \tag{2}$$

where $r \in \langle 0; \pi\rangle$ and

$$\boldsymbol{n}_1 = \frac{\boldsymbol{x}_1}{\|\boldsymbol{x}_1\|} \qquad\qquad \boldsymbol{n}_2 = \frac{x_2}{\|x_2\|}\,. \tag{3}$$

The distance r in (2) is measured in radians. When the sphere has a radius equal to one, the computed distance in radians is equal to the distance measured along the surface of the sphere.

The RBF interpolation on a sphere is computed using the same formula as standard RBF. The only difference compared to the standard equation for RBF interpolation is when computing the distance between two points, as both of these approaches can be used.

2.1 Example of Vector Field on Sphere on Synthetic data

An example of a vector field on a sphere can be described analytically. This analytical description must fulfill one criteria, which is that this function is continuous all over the sphere. For this purpose, we can use goniometric functions that have a period equal to 2π, i.e.

$$\sin\alpha = \sin(\alpha + k \cdot 2\pi) \qquad \cos\alpha = \cos(\alpha + k \cdot 2\pi)\,, \tag{4}$$

where k is an integer, i.e. $k \in \mathbb{Z}$.

The first example of a vector field on a sphere is described using the following equations:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sin 4\delta \\ \cos 4\theta \end{bmatrix} \qquad \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sin 3\delta + \cos 4\delta \cdot \cos 3\delta \\ \cos 4\theta - \sin 4\theta \cdot \sin 3\delta \end{bmatrix}. \tag{5}$$

where δ is an azimuth angle, i.e. $\delta \in (-\pi; \pi\rangle$ and θ is a zenith angle, i.e. $\theta \in \langle 0; \pi\rangle$. Data $[u, v]^T$ represents the direction vector on the surface of the sphere at point $[P_x, P_y, P_z]^T$:

$$[P_x \quad P_y \quad P_z]^T = [\sin\theta\cos\delta \quad \sin\theta\sin\delta \quad \cos\theta]^T. \tag{6}$$

The vector fields (5) were discretized on uniformly distributed 10 000 points on the sphere and then interpolated using RBF on the sphere with CSRBF with a shape parameter equal to 1:

$$\varphi(r) = (1-r)_+^4(4r+1)\,. \tag{7}$$

The interpolation, when using (2) to compute the distance r for basis function $\varphi(r)$, can be seen in Figure 2(a, b). This visualization was created with ray-tracing and line integral convolution on the sphere.

To measure the quality of the interpolation, we can compute the mean error of speed and the mean error of angular displacement of vectors. The mean errors were computed for 10^6 randomly generated positions on the sphere. The results for both equations (5) and both ways of calculating the distance between two points can be seen in Table 1. Note that both vectors $[u, v]^T$ in (5) are computed in $[ms^{-1}]$.

Table 1. Errors of RBF interpolated vector fields (5) on a sphere for both ways of computing distance between two points

		Speed error $[ms^{-1}]$	Angular displacement error $[rad]$
Euclidian distance	vector field (5 left)	$2.452 \cdot 10^{-4}$	$4.233 \cdot 10^{-4}$
	vector field (5 right)	$1.884 \cdot 10^{-3}$	$2.672 \cdot 10^{-3}$
Spherical distance	vector field (5 left)	$1.686 \cdot 10^{-4}$	$3.074 \cdot 10^{-4}$
	vector field (5 right)	$1.379 \cdot 10^{-3}$	$1.906 \cdot 10^{-3}$

It can be seen that the RBF interpolation when using spherical distance gives better results for both vector fields, i.e. more accurate speed and more accurate orientation at every location on the sphere on average, see Table 1. The RBF interpolation is less accurate for the vector field (5 right) than for the vector field (5 left). The reason is that the vector field (5 right) is significantly more complicated than (5 left). The distribution of speed errors and angular displacement errors is visualized in Figure 1. Histograms were created from 10^6 samples and data were grouped into 71 bins.

2.2 Real Example of Vector Field on Sphere on Experiment Data

Numerical forecasts can predict weather as well as wind velocity and direction. For this example, one such prediction of the wind vector field for the whole world [US GFS global weather model] was used. This data contains information about wind speed and wind direction every one degree in latitude and longitude. Therefore, the resolution of the numerically computed dataset is 360×180, which is 64 800 vectors in total.

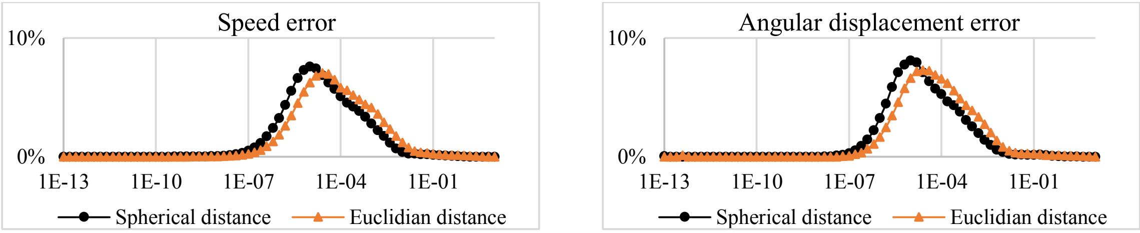
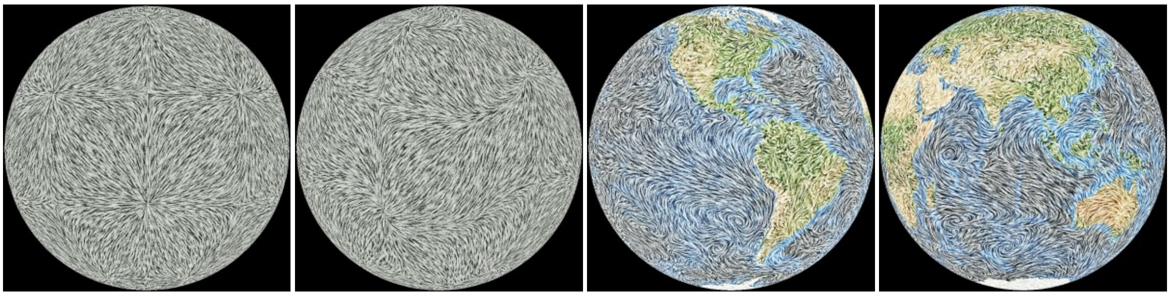


Figure 1. Histogram of speed error distribution (a) and displacement error distribution (b) for a vector field (5 left).



a) b) c) d)

Figure 2. Visualization of examples of vector fields. All vector fields were interpolated using RBF and visualized as LIC images on a sphere. Equations (5 left) (a) and (5 right) (b). Sources, resp. sinks, and saddles are clearly seen in both images . Both images (a, b) are visually identical to the ones with an original analytical description. Visualization of an RBF interpolated wind vector field from numerical simulation (c, d).

Some reduction of this dataset was done, as for the North or South Pole only one vector is needed and for locations near these two poles, the computed vectors can be reduced as well. After the reduction, there were $62\,742$ vectors. This wind data were interpolated using RBF with CSRBF (7) and shape parameter $\varepsilon = 1$. The RBF interpolation was used to create the visualization of wind vector field on the sphere, Figure 2(c, d).

CONCLUSION

Two approaches for interpolation on a sphere using Radial Basis Functions were presented. The new approach uses the spherical distance as the parameter for the radial basis function computation. The proposed approach gives better results for interpolation on a sphere in comparison to the original standard approach using the Euclidian distance. The proposed method was verified on synthetic analytical datasets and non-trivial real wind datasets of a weather forecast [US GFS global weather model]. In future, the proposed approach will be explored more deeply for t-varying datasets together with aspects of implementation for very large dataset processing.

ACKNOWLEDGEMENT

The authors would like to thank their colleagues at the University of West Bohemia, Plzen, for their discussions and suggestions, and anonymous reviewers for the valuable comments and suggestions they provided. The research was supported by MSMT CR projects LH12181 and SGS 2016-013.

REFERENCES

Aguilar, F. J., et al., 2005. Effects of terrain morphology, sampling density, and interpolation methods on grid DEM accuracy. *Photogrammetric Engineering & Remote Sensing*, Vol. 71, No. 7, pp. 805-816.

Baxter, B. J., Hubbert, S., 2001. Radial basis functions for the sphere. *Recent Progress in Multivariate Approximation*, pp. 33-47, Birkhäuser Basel.

Eldrandaly, K. A., Abu-Zaid, M. S., 2011. Comparison of Six GIS-Based Spatial Interpolation Methods for Estimating Air Temperature in Western Saudi Arabia. *Journal of Environmental Informatics*, Vol. 18, No. 1.

Fasshauer, G.E., 2007. *Meshfree Approximation Methods with MATLAB*. World Scientific Publ. Co., Inc., NJ, USA.

Flyer, N. et al., 2014. Radial basis function-generated finite differences: A mesh-free method for computational geosciences. *Handbook of Geomathematics*. Springer, Berlin.

Golitschek, M. V., Light, W. A., 2001. Interpolation by polynomials and radial basis functions on spheres. *Constructive Approximation*, Vol. 17, No. 1, pp. 1-18.

Pan, R. and Skala, V., 2011. A Two-level Approach to Implicit Surface Modeling with Compactly Supported Radial Basis Functions. *In Eng. Comput. (Lond.)*, Vol. 27, No. 3, pp. 299-307.

Turk, G., O'Brien, J. F., 2002. Modelling with implicit surfaces that interpolate. *ACM Transactions on Graphics*, Vol. 21, No. 4, pp. 855-873.

US GFS global weather model. *National Centers for Environmental Information*, https://www.ncdc.noaa.gov/data-access/model-data/model-datasets/global-forcast-system-gfs, [downloaded: 13.1.2016].